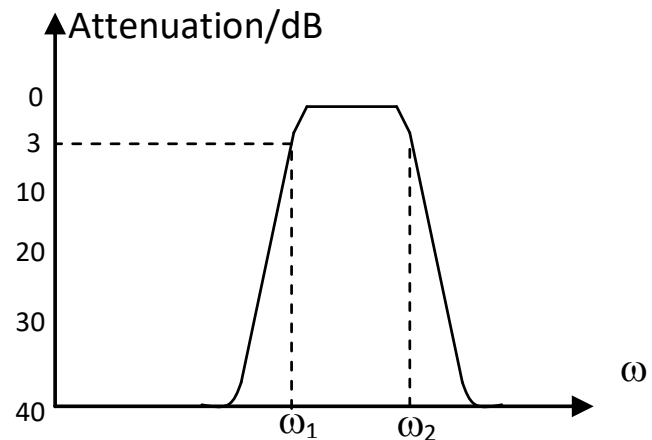
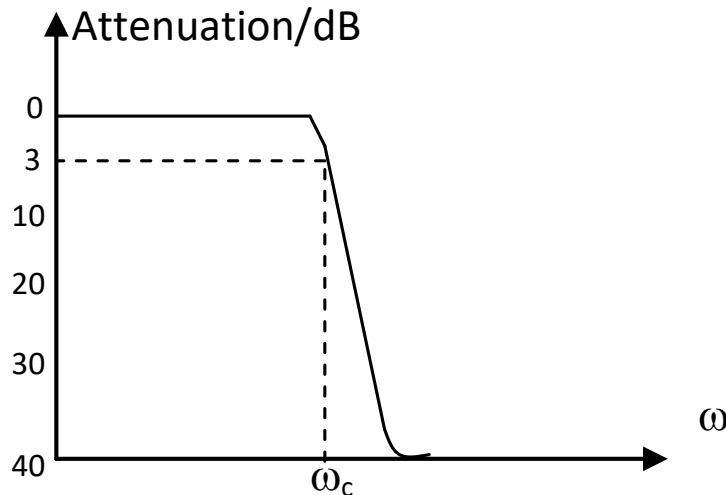


MICROWAVE FILTERS

What is a Microwave filter ?

- Circuit that controls the frequency response at a certain point in a microwave system
- provides perfect transmission of signal for frequencies in a certain passband region
- infinite attenuation for frequencies in the stopband region



FILTER DESIGN METHODS

Filter Design Methods

Two types of commonly used design methods:

- **Image Parameter Method**
- **Insertion Loss Method**

• Image parameter method yields a usable filter

Filters designed using the *image parameter method* consist of a ***cascade of simpler two port filter*** sections to provide the desired cutoff frequencies and attenuation characteristics but do not allow the specification of a particular frequency **response** over the complete operating range. Thus, although the procedure is relatively simple, the design of filters by the image parameter method often must be **iterated many times** to achieve the desired results.

Filter Design by The Insertion Loss Method

The **perfect filter** would have **zero insertion loss** in the **pass-band**, **infinite attenuation** in the **stop-band**, and a **linear phase response** in the pass-band.

Characterization by Power Loss Ratio

$$\text{Power Loss Ratio} = P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

If both load and source are matched. $P_{LR} = \frac{1}{|S_{12}|^2}$

The insertion loss (IL) is dB is $IL = 10 \log P_{LR}$

Filter Design by The Insertion Loss Method

Characterization by Power Loss Ratio

$$\Gamma(\omega) = \frac{Z(\omega) - Z_o}{Z(\omega) + Z_o} = \frac{R(\omega) - Z_o + jX(\omega)}{R(\omega) + Z_o + jX(\omega)}$$

$$Z(\omega) = R(\omega) + jX(\omega)$$

$$\Gamma(-\omega) = \frac{R(\omega) - Z_o - jX(\omega)}{R(\omega) + Z_o - jX(\omega)} = \Gamma^*(\omega)$$

$$|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega) = \Gamma(\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2 \quad (\text{Even functions of } \omega)$$

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

Where **M** and **N** are real polynomials in ω^2 .

$$P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2} \rightarrow P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

The magnitude of the voltage gain

Filter Design by The Insertion Loss Method

Proof

$$\Gamma(\omega) = \frac{R(\omega) - Z_o + jX(\omega)}{R(\omega) + Z_o + jX(\omega)} \quad \Gamma^*(\omega) = \frac{R(\omega) - Z_o - jX(\omega)}{R(\omega) + Z_o - jX(\omega)}$$

$$|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega)$$

$$\Gamma(\omega)\Gamma^*(\omega) = \left(\frac{R(\omega) - Z_o + jX(\omega)}{R(\omega) + Z_o + jX(\omega)} \right) \left(\frac{R(\omega) - Z_o - jX(\omega)}{R(\omega) + Z_o - jX(\omega)} \right)$$

$$\Gamma(\omega)\Gamma^*(\omega) = \frac{(R(\omega) - Z_o)^2 + X^2(\omega)}{(R(\omega) + Z_o)^2 + X^2(\omega)}$$

$$\Gamma(\omega)\Gamma^*(\omega) = \frac{(R(\omega) - Z_o)^2 + X^2(\omega)}{(R(\omega) + Z_o)^2 + X^2(\omega) - (R(\omega) - Z_o)^2 + (R(\omega) - Z_o)^2}$$

$$\Gamma(\omega)\Gamma^*(\omega) = \frac{(R(\omega) - Z_o)^2 + X^2(\omega)}{(R(\omega) - Z_o)^2 + X^2(\omega) + 4R(\omega)Z_o}$$

Filter Design by The Insertion Loss Method

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

Maximally Flat

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

Equal Ripple

$$P_{LR} = 1 + k^2 T_N^2 \left(\frac{\omega}{\omega_c} \right)$$

Linear Phase

$$\phi(\omega) = A\omega \left[1 + p \left(\frac{\omega}{\omega_c} \right)^{2N} \right]$$

Filter Design by The Insertion Loss Method

Practical filter response:

Maximally flat:

- also called the binomial or Butterworth response,
- is optimum in the sense that it provides the **flattest** possible **passband** response for a given filter complexity.

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^N$$

Equal ripple also known as Chebyshev.

- sharper cutoff

$$P_{LR} = 1 + k^2 T_N^2(\omega)$$

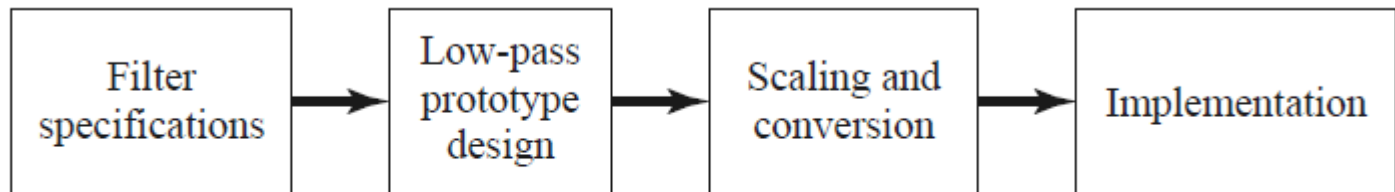
ω – frequency of filter

ω_c – cutoff frequency of filter

N – order of filter

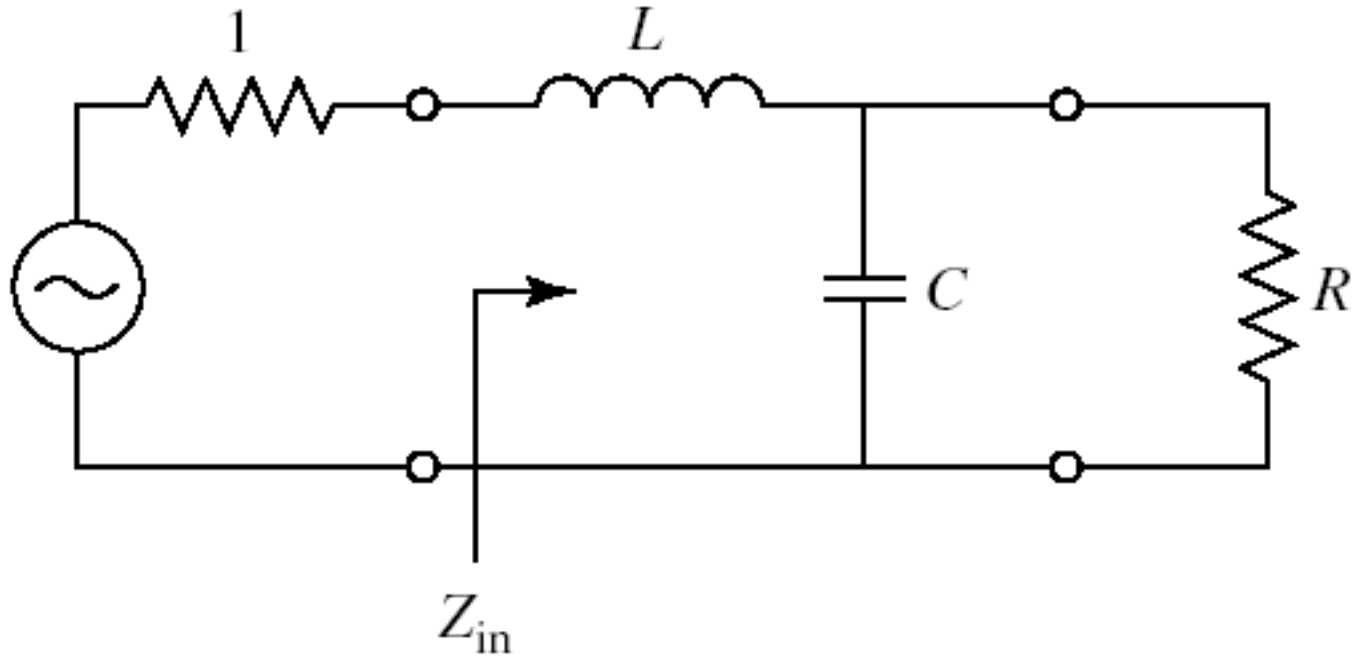
Filter Design by The Insertion Loss Method

- The insertion loss method (ILM) allows a systematic way to design and synthesize a filter with various frequency response.
- Design usually begins by designing a normalized low-pass prototype (LPP). The LPP is a low-pass filter with source and load resistance of 1Ω and cutoff frequency of 1 Radian/s.
- Impedance transformation and frequency scaling are then applied to denormalize the LPP and synthesize different type of filters with different cutoff frequencies.



The process of filter design by the insertion loss method.

Low Pass Filter Prototype



Low pass filter prototype, $N = 2$

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

$$\text{for } N = 2, \quad P_{LR} = 1 + \omega^4.$$

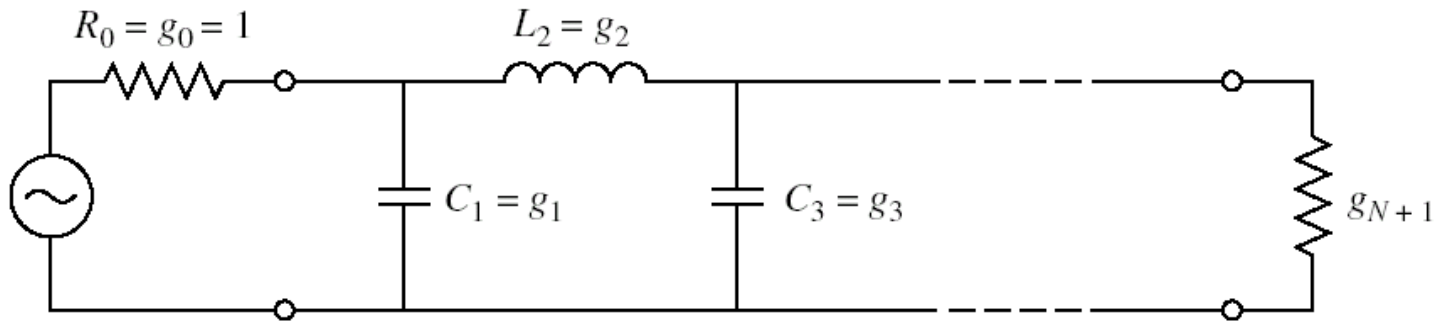
$$P_{LR} = \frac{1}{1 - |\Gamma|^2} \quad \Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$

$$P_{LR} = 1 + \frac{1}{4R} [(1 - R)^2 + (R^2 C^2 + L^2 - 2LCR^2)\omega^2 + L^2 C^2 R^2 \omega^4].$$

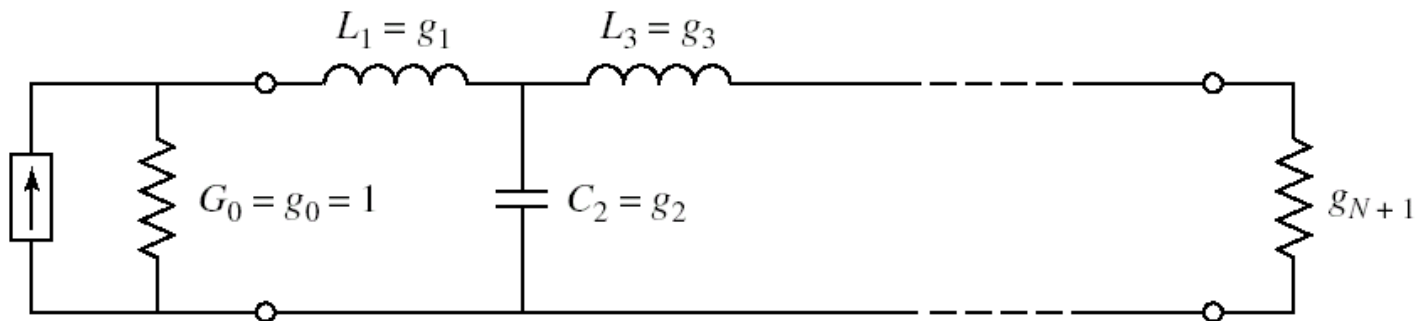
$$L = C = \sqrt{2}. = 1.4142$$

Low Pass Filter Prototype – Ladder Circuit

For an arbitrary number of elements N .



(a)



(b)

Ladder circuit for low pass filter prototypes and their element definitions. (a) begin with shunt element. (b) begin with series element.

g_0 = generator resistance, generator conductance.

g_k = inductance for series inductors, capacitance for shunt capacitors.

($k=1$ to N)

g_{N+1} = load resistance if g_N is a shunt capacitor, load conductance if g_N is a series inductor.

Element Values for Maximally-Flat LPF Prototype

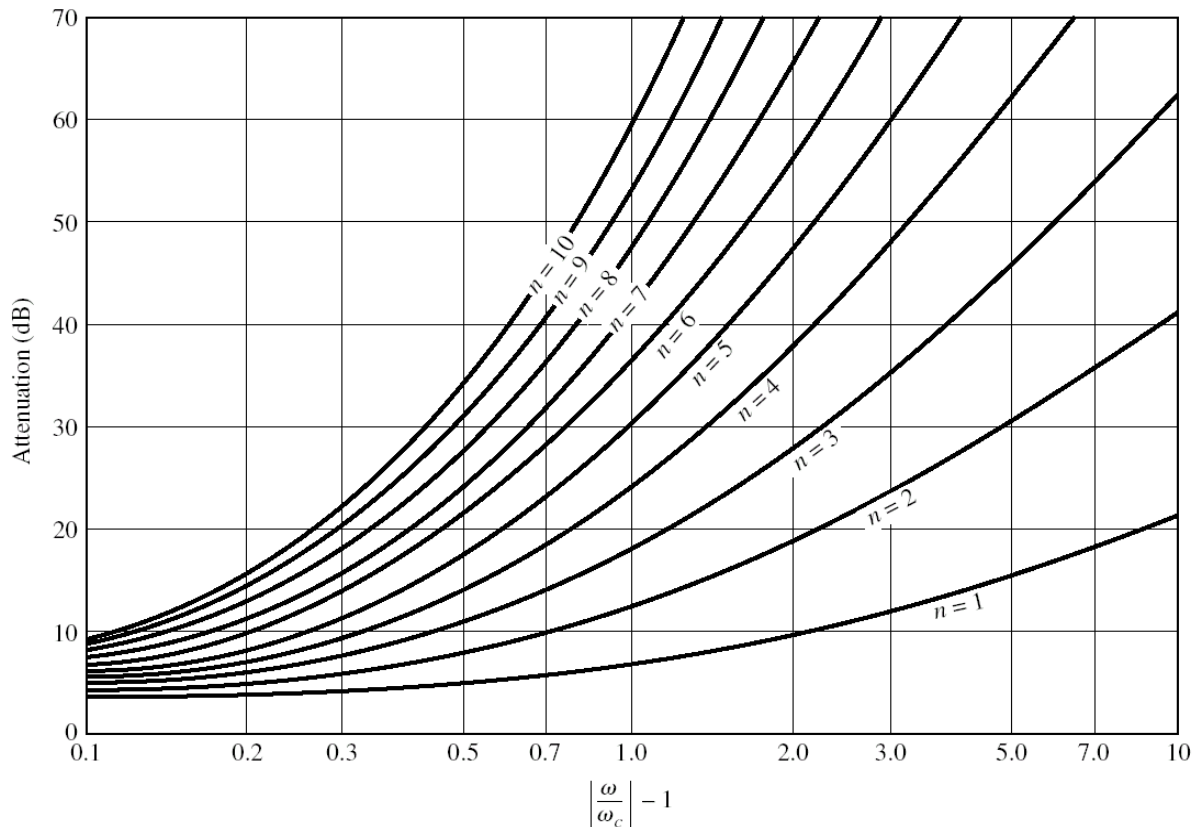
($g_0 = 1, \omega_c = 1, N = 1 \text{ to } 10$)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Element values for maximally flat LPF prototypes

Low Pass Filter Prototype – Maximally Flat

To determine the order (size) of the filter. Is usually determined by the **specification of the insertion loss at some frequency in the stopband** of the filter.



Attenuation versus normalized frequency for maximally flat filter prototypes.

Low Pass Filter Prototype – Equal Ripple

For an equal ripple low pass filter with a cutoff frequency $\omega_c =$
The power loss ratio is:

$$P_{LR} = 1 + k^2 T_N^2(\omega)$$

Element Values for Equal-Ripple LPF Prototype

($g_0 = 1, \omega_c = 1, N = 1 \text{ to } 10, 0.5\text{dB ripple}$)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

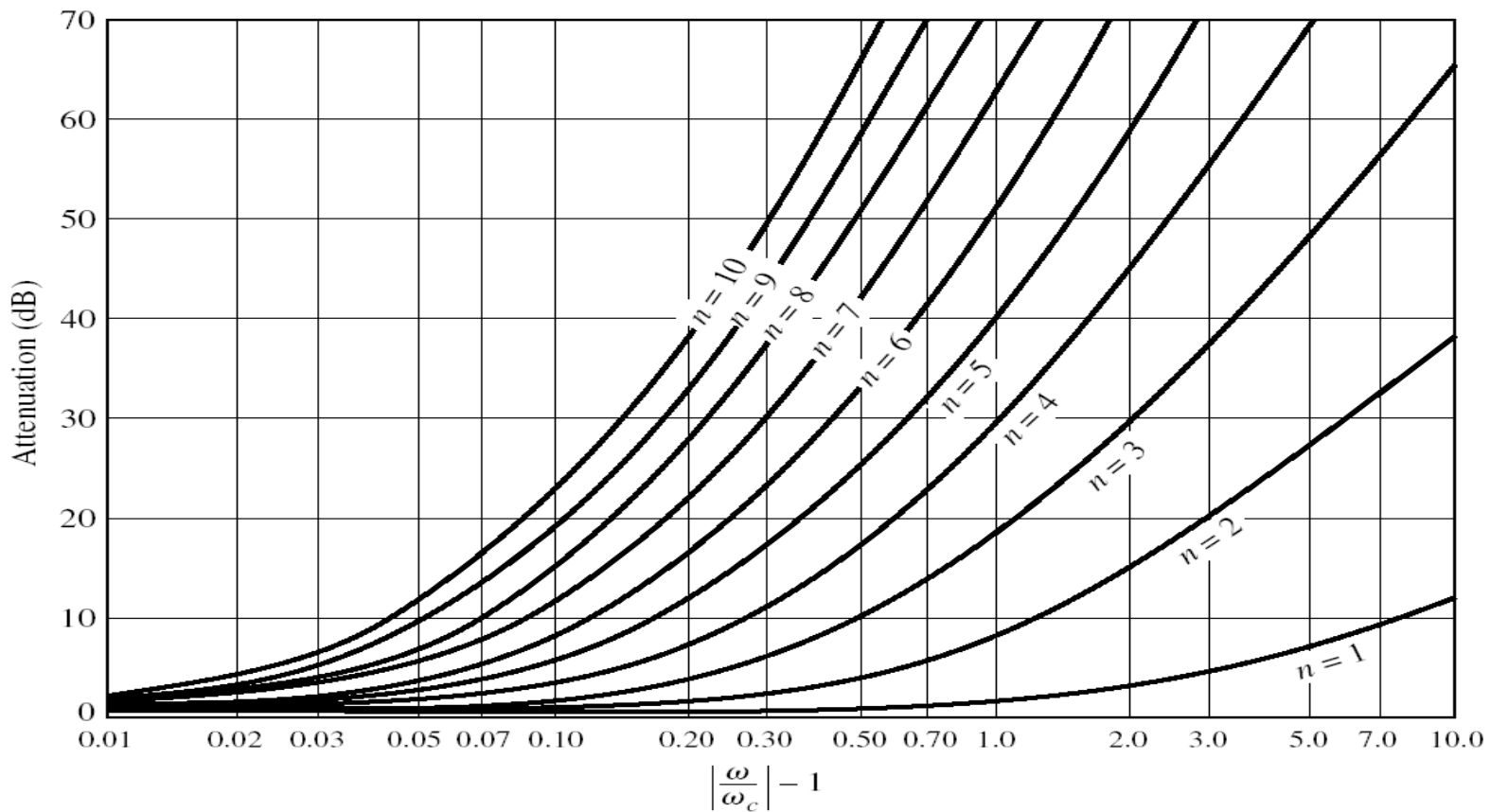
Element values for equal ripple LPF prototypes (0.5 dB ripple level)

THE INSERTION LOSS METHOD

Element Values for Equal-Ripple LPF Prototype
 ($g_0 = 1, \omega_c = 1, N = 1 \text{ to } 10, 3.0\text{dB ripple}$)

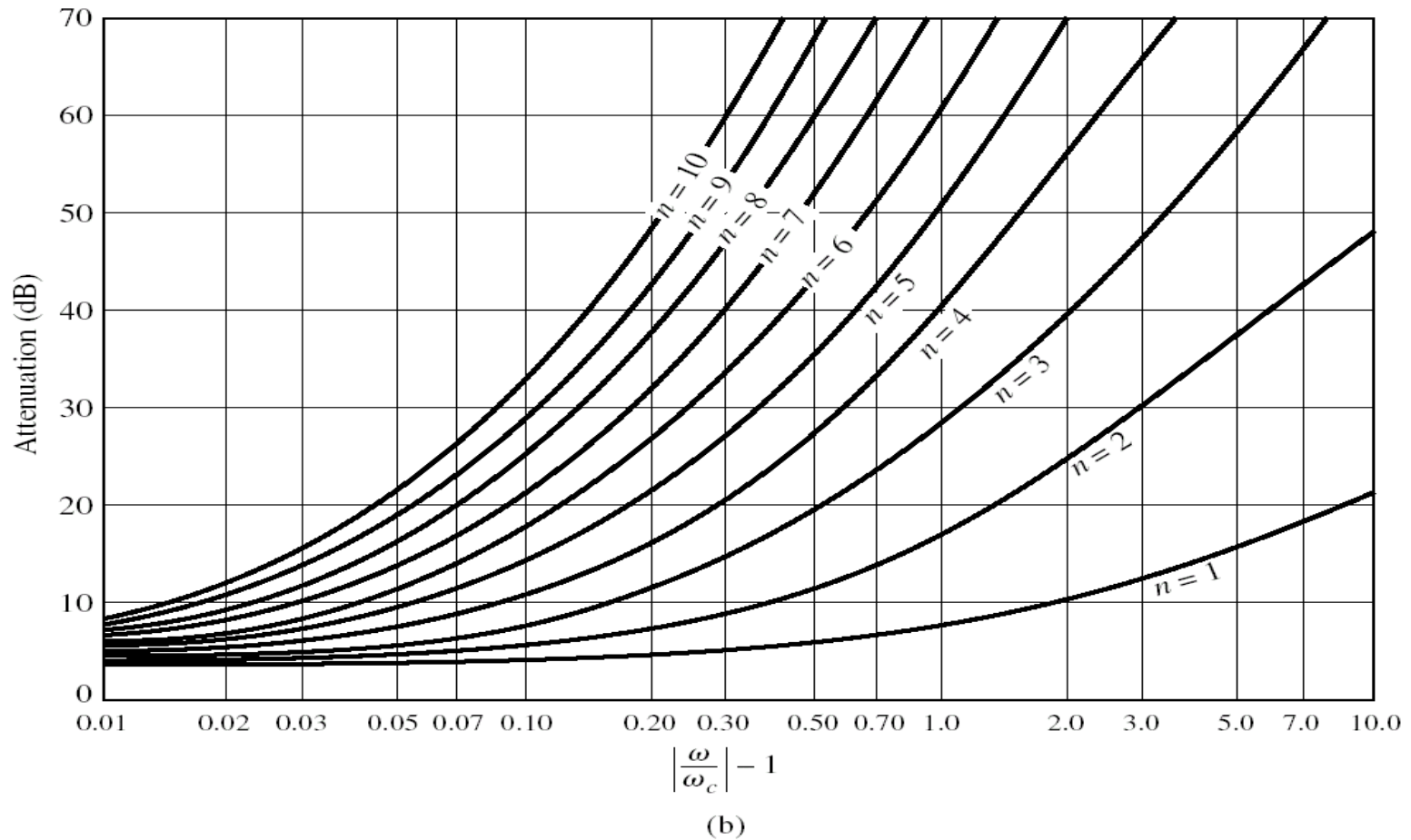
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Element values for equal ripple LPF prototypes (3.0 dB ripple level).



(a)

Attenuation versus normalized frequency for equal-ripple filter prototypes.
(0.5 dB ripple level)



Attenuation versus normalized frequency for equal-ripple filter prototypes
(3.0 dB ripple level)

Scaling and **TRANSFORMATIONS**

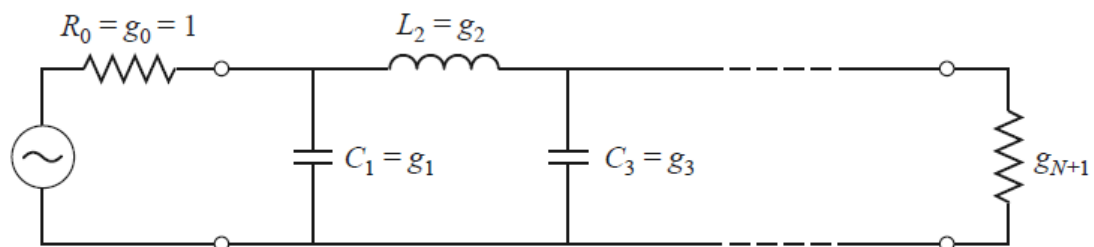
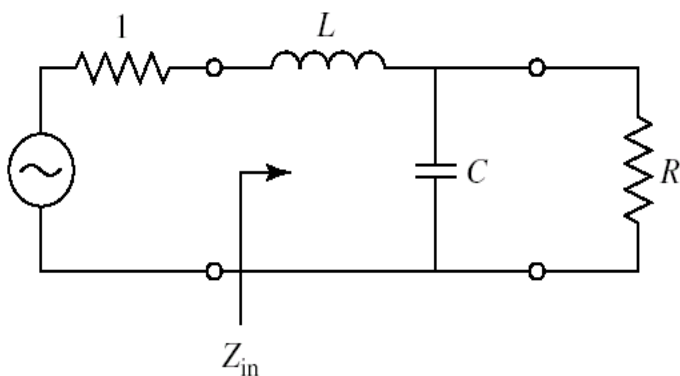
Scaling

Example 1: LPF

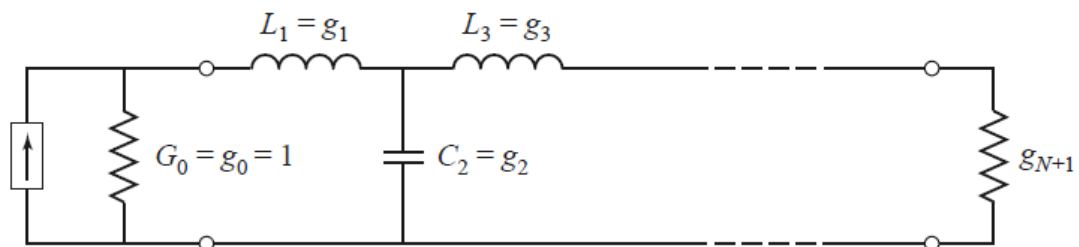
$$L = C = \sqrt{2} = 1.4142$$

for $N = 2$,

N	g_1	g_2	g_3	g_4
1	2.0000	1.0000		
2	1.4142	1.4142	1.0000	
3	1.0000	2.0000	1.0000	1.0000



(a)



Low Pass Filter Prototype – Impedance Scaling

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R'_s = R_0$$

$$R'_L = R_0 R_L$$

Frequency scaling for the low pass filter:

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

The new element values of the prototype filter:

$$jX_k = j \frac{\omega}{\omega_c} L_k = j\omega L'_k$$

$$jB_k = j \frac{\omega}{\omega_c} C_k = j\omega C'_k$$

The new element values are given by:

$$L'_k = \frac{L_k}{\omega_c}$$
$$C'_k = \frac{C_k}{\omega_c}$$

finally

The new element values are given by:

$$L'_k = \frac{R_0 L_k}{\omega_c}$$

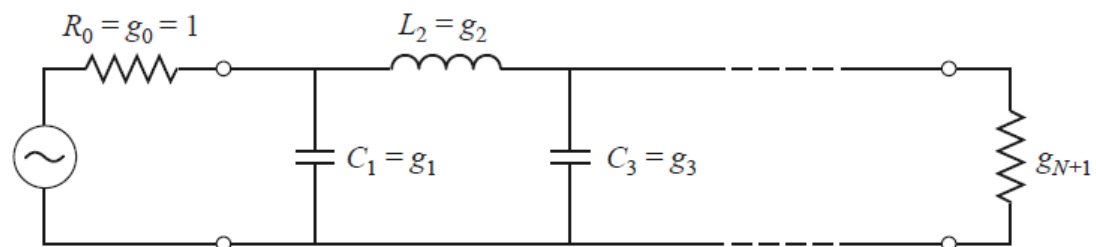
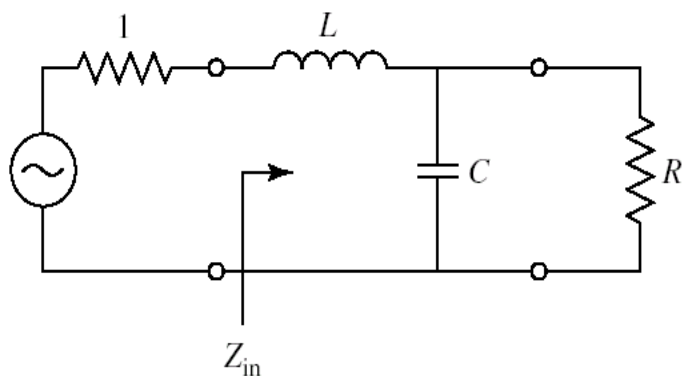
$$C'_k = \frac{C_k}{R_0 \omega_c}$$

Example 1: LPF

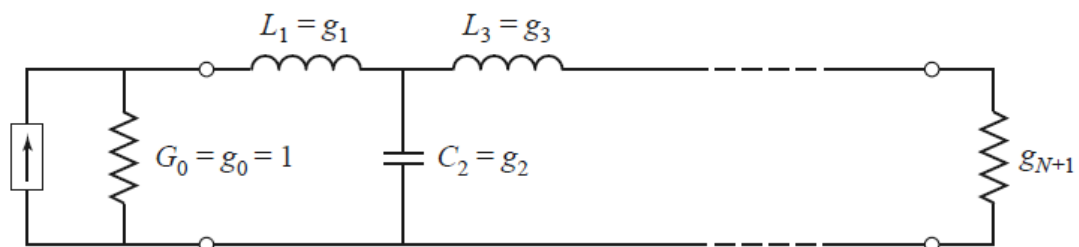
$$L = C = \sqrt{2} = 1.4142$$

for $N = 2$,

N	g_1	g_2	g_3	g_4
1	2.0000	1.0000		
2	1.4142	1.4142	1.0000	
3	1.0000	2.0000	1.0000	1.0000

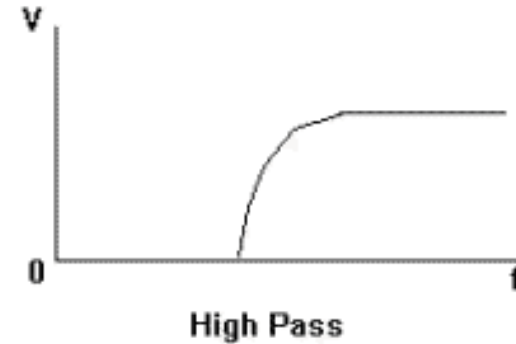
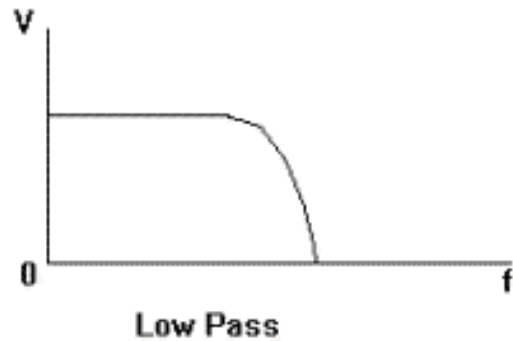


(a)



TRANSFORMATIONS

Low-pass to high-pass transformation



$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

$$jX_k = -j\frac{\omega_c}{\omega}L_k = \frac{1}{j\omega C'_k},$$

$$jB_k = -j\frac{\omega_c}{\omega}C_k = \frac{1}{j\omega L'_k},$$

$$C'_k = \frac{1}{\omega_c L_k},$$

$$L'_k = \frac{1}{\omega_c C_k}.$$



$$C'_k = \frac{1}{R_0 \omega_c L_k},$$

$$L'_k = \frac{R_0}{\omega_c C_k}.$$

Low pass to high pass transformation

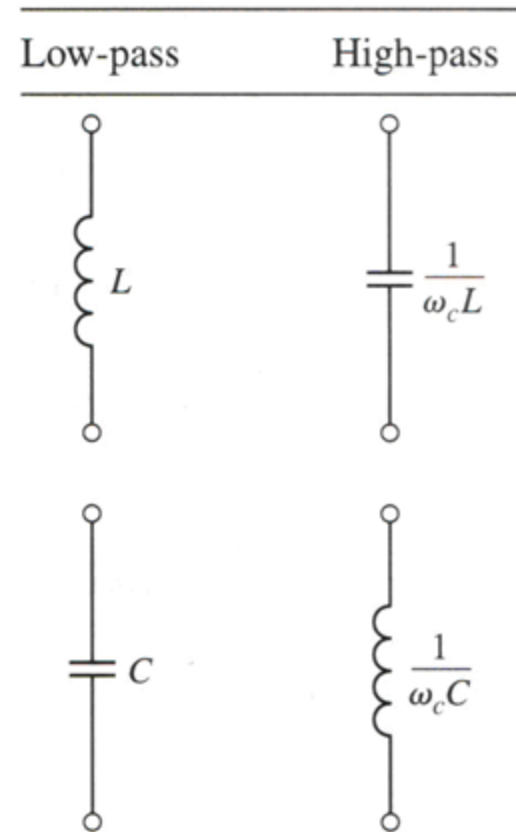
The frequency substitution:

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

The new component values are given by:

$$C'_k = \frac{1}{R_0 \omega_c L_k}$$

$$L'_k = \frac{R_0}{\omega_c C_k}$$



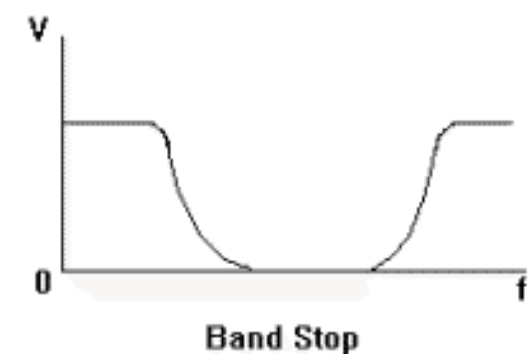
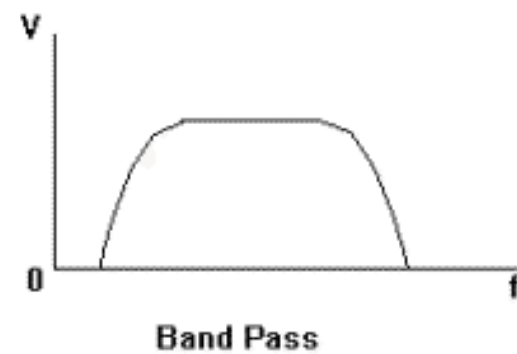
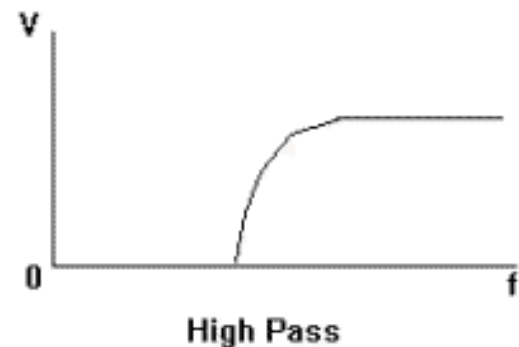
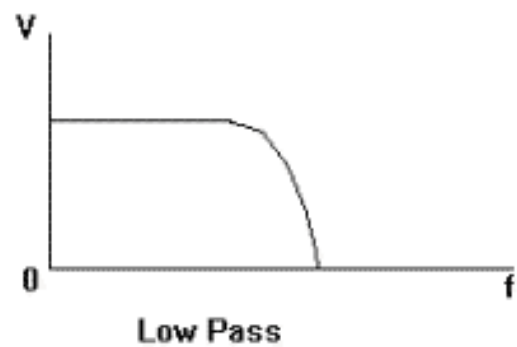


Fig. 3

Low pass to Bandpass transformation

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{\Delta} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)$$

Where,

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

The center frequency is:

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

The series inductor, L_k , is transformed to a series LC circuit with element values:

$$L'_k = \frac{L_k}{\Delta \omega_0}$$

$$C'_k = \frac{\Delta}{\omega_0 L_k}$$

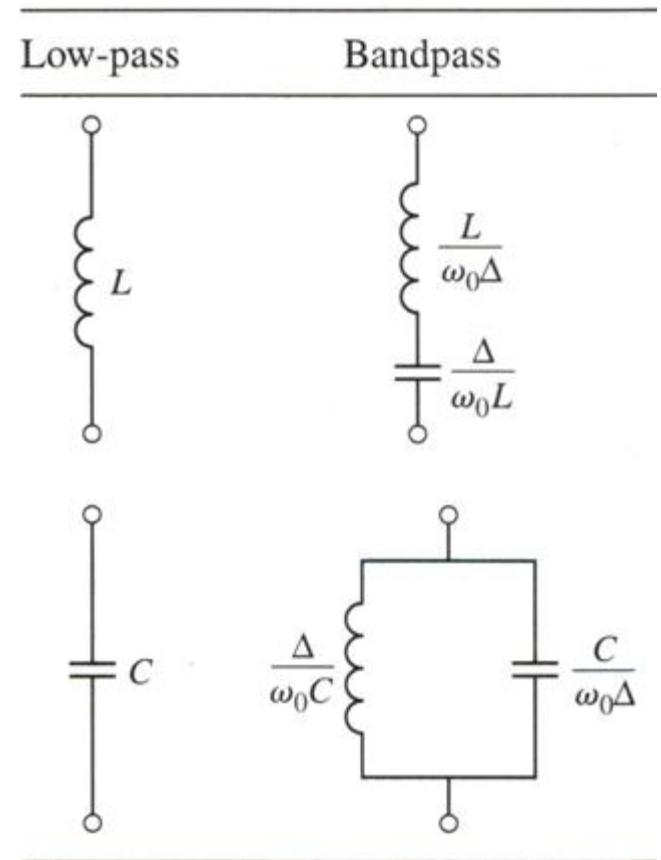
The shunt capacitor, C_k , is transformed to a shunt LC circuit with element values:

$$L'_k = \frac{\Delta}{\omega_0 C_k}$$

$$C'_k = \frac{C_k}{\Delta \omega_0}$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$



Low pass to Bandstop transformation

$$\omega \leftarrow -\Delta \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

Where,

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

The center frequency is:

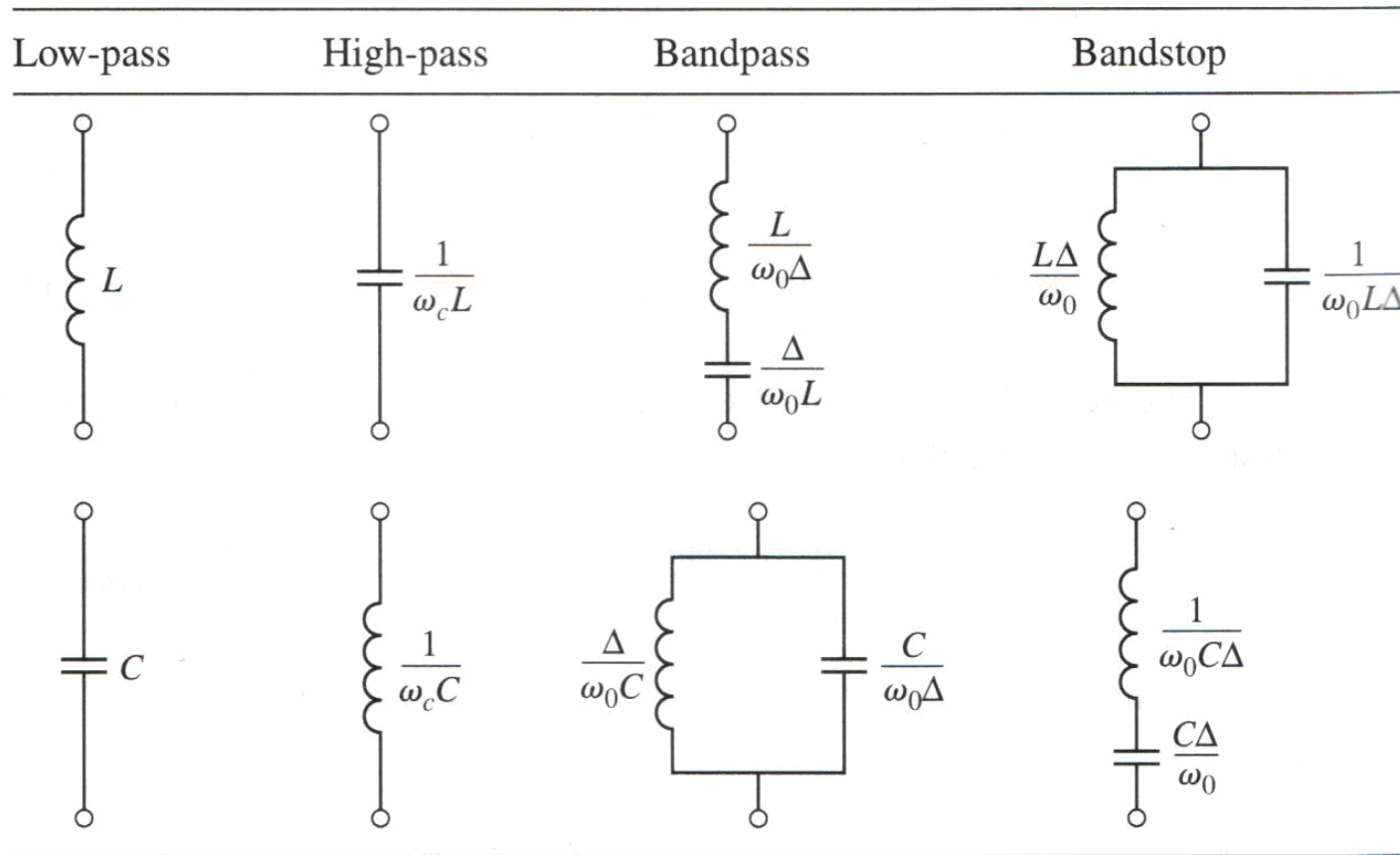
$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

The series inductor, L_k , is transformed to a parallel LC circuit with element values:

$$L'_k = \frac{\Delta L_k}{\omega_0}$$
$$C'_k = \frac{1}{\omega_0 \Delta L_k}$$

The shunt capacitor, C_k , is transformed to a series LC circuit with element values:

$$L'_k = \frac{1}{\omega_0 \Delta C_k}$$
$$C'_k = \frac{\Delta C_k}{\omega_0}$$



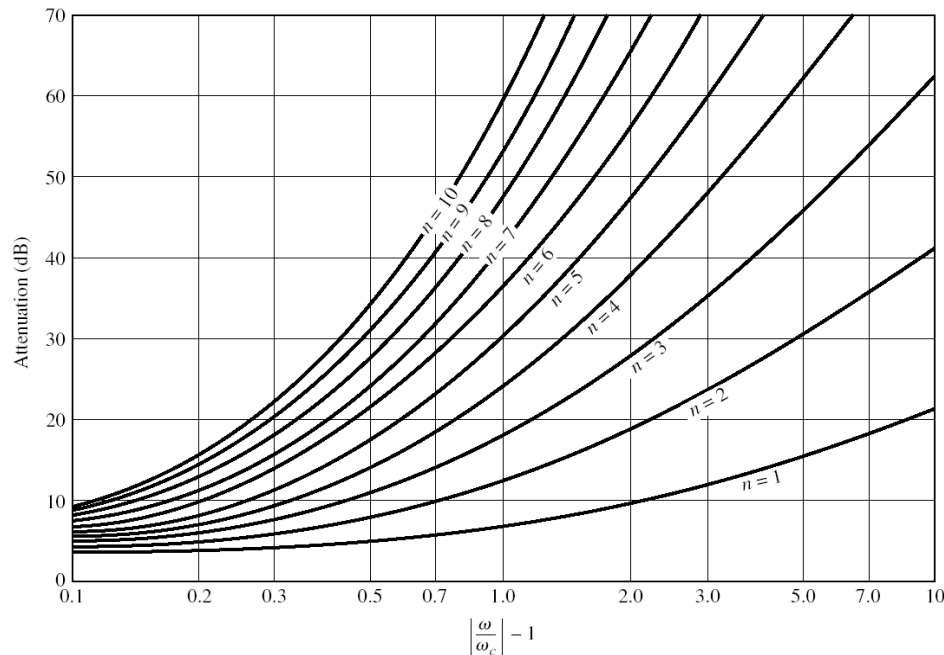
Example 2:

Design a maximally flat **low pass** filter with a cutoff freq of 2 GHz, impedance of 50 Ω , and at least 15 dB insertion loss at 3 GHz. Compute and compare with an equal-ripple (3.0 dB ripple) having the same order.

Solution:

First find the order of the maximally flat filter to satisfy the insertion loss specification at 3 GHz.

We can find the normalized freq by using: $\left| \frac{\omega}{\omega_c} \right| - 1 = \left| \frac{3}{2} \right| - 1 = 0.5$



Element Values for Maximally-Flat LPF Prototype

($g_0 = 1, \omega_c = 1, N = 1 \text{ to } 10$)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

$$g_1 = 0.618$$

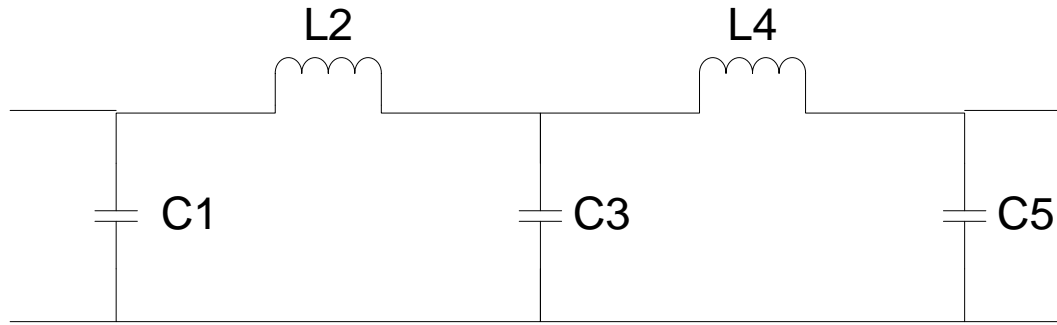
$$g_2 = 1.618$$

$$g_3 = 2.0$$

$$g_4 = 1.618$$

$$g_5 = 0.618$$

The ladder diagram of the LPF prototype to be used is as follow:



$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R'_s = R_0$$

$$R'_L = R_0 R_L$$

$$C_1 = \frac{g_1}{R_0 \omega_c}$$

$$L_2 = \frac{R_0 g_2}{\omega_c}$$

$$C_3 = \frac{g_3}{R_0 \omega_c}$$

$$L_4 = \frac{R_0 g_4}{\omega_c}$$

$$C_5 = \frac{g_5}{R_0 \omega_c}$$

LPF prototype for maximally flat filter

$$C_1 = \frac{g_1}{R_0 \omega_c} = \frac{0.618}{(50)(2\pi \times 2 \times 10^9)} = 0.984 \text{ pF}$$
$$L_2 = \frac{R_0 g_2}{\omega_c} = \frac{50 \times 1.618}{(2\pi \times 2 \times 10^9)} = 6.438 \text{ nH}$$
$$C_3 = \frac{g_3}{R_0 \omega_c} = \frac{2.00}{(50)(2\pi \times 2 \times 10^9)} = 3.183 \text{ pF}$$
$$L_4 = \frac{R_0 g_4}{\omega_c} = \frac{50 \times 1.618}{(2\pi \times 2 \times 10^9)} = 6.438 \text{ nH}$$
$$C_5 = \frac{g_5}{R_0 \omega_c} = \frac{0.618}{(50)(2\pi \times 2 \times 10^9)} = 0.984 \text{ pF}$$

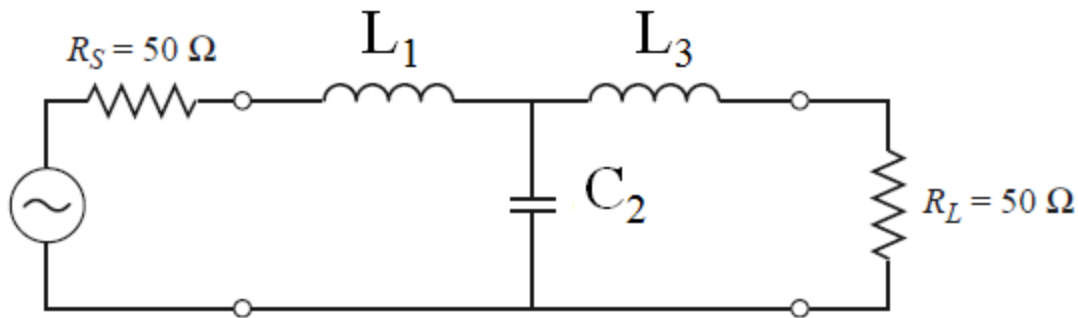
LPF prototype for **equal** ripple filter:

$$\begin{aligned}g_1 &= 3.4817 & C_1 &= \frac{g_1}{R_0 \omega_c} = \frac{3.4817}{(50)(2\pi \times 2 \times 10^9)} = 5.541 \text{ pF} \\g_2 &= 0.7618 & L_2 &= \frac{R_0 g_2}{\omega_c} = \frac{50 \times 0.7618}{(2\pi \times 2 \times 10^9)} = 3.031 \text{ nH} \\g_3 &= 4.5381 & C_3 &= \frac{g_3}{R_0 \omega_c} = \frac{4.5381}{(50)(2\pi \times 2 \times 10^9)} = 7.223 \text{ pF} \\g_4 &= 0.7618 & L_4 &= \frac{R_0 g_4}{\omega_c} = \frac{50 \times 0.7618}{(2\pi \times 2 \times 10^9)} = 3.031 \text{ nH} \\g_5 &= 3.4817 & C_5 &= \frac{g_5}{R_0 \omega_c} = \frac{3.4817}{(50)(2\pi \times 2 \times 10^9)} = 5.541 \text{ pF}\end{aligned}$$

Example:

Design a **band pass** filter having a 0.5 dB equal-ripple response, with $N = 3$. The center frequency is 1 GHz, the bandwidth is 10%, and the impedance is 50 Ω .

Solution: The low pass filter (LPF) prototype ladder diagram is shown as follow:



$$g_1 = 1.5963 = L_1$$

$$g_2 = 1.0967 = C_2$$

$$g_3 = 1.5963 = L_3$$

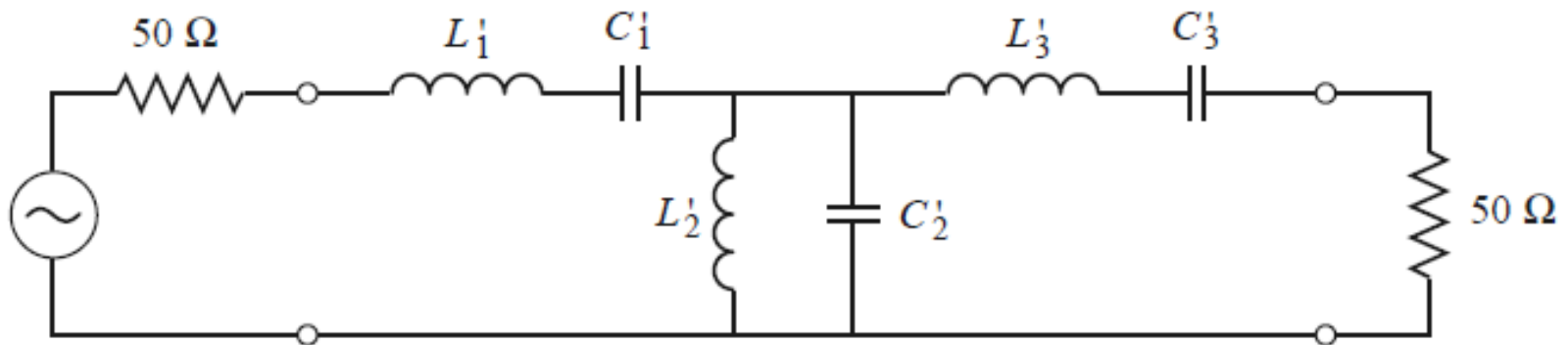
$$g_4 = 1.000 = R_L$$

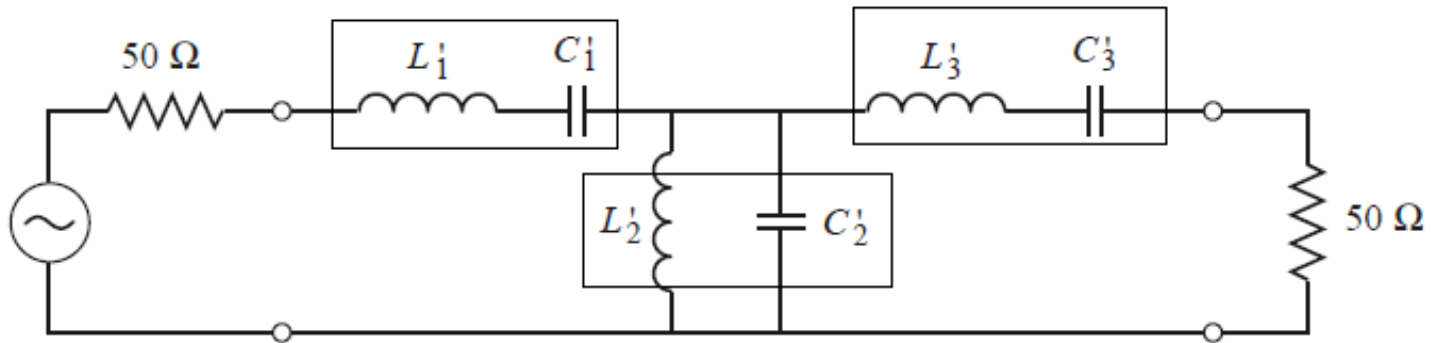
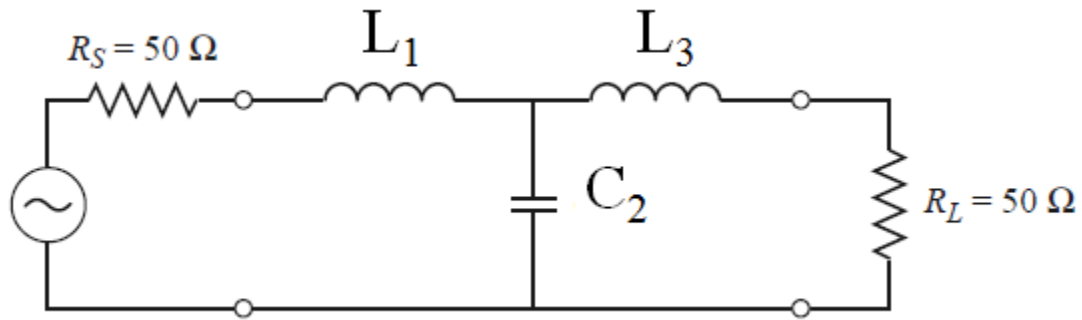
Transforming the LPF prototype to the **BPF** prototype

$$\Delta = 0.1$$

$$N = 3$$

$$\omega = 1 \text{ GHz}$$





$$L_1' = \frac{L_1 Z_0}{\omega_0 \Delta} = \frac{1.5963 \times 50}{(2\pi \times 1 \times 10^9 \times 0.1)} = 127.0 \text{ nH}$$

$$C_1' = \frac{\Delta}{Z_0 \omega_0 L_1} = \frac{0.1}{(50)(2\pi \times 2 \times 10^9 \times 1.5963)} = 0.199 \text{ pF}$$

$$L_2' = \frac{\Delta Z_0}{\omega_0 C_2} = \frac{0.1 \times 50}{(2\pi \times 1 \times 10^9 \times 1.0967)} = 0.726 \text{ nH}$$

$$C_2' = \frac{C_2}{\omega_0 \Delta Z_0} = \frac{1.0967}{(2\pi \times 1 \times 10^9)(0.1)(50)} = 34.91 \text{ pF}$$

$$L'_3 = \frac{L_3 Z_0}{\omega_0 \Delta} = \frac{1.5963 \times 50}{(2\pi \times 1 \times 10^9 \times 0.1)} = 127.0 \text{ nH}$$

$$C'_3 = \frac{\Delta}{Z_0 \omega_0 L_3} = \frac{0.1}{(50)(2\pi \times 2 \times 10^9 \times 1.5963)} = 0.199 \text{ pF}$$

Thank you